Evolution of coherence, concurrence and Fisher information under global environmental interaction

Subhashish Banerjee

Indian Institute Of Technology, Jodhpur (Rajasthan)

(ICQF16, NIT, Patna)

Overview of Talk

- We make use of a global system-bath interaction, both in the absence as well as presence of non-Markovian noise, to study the impact of non-Markovianity on some facets of quantumness in the system.
- We try to understand the impact of this interaction on the ubiquitous decoherence.

A Brief Preview and Motivation

- Due to the advent of quantum technologies such as quantum communication, quantum cryptography, there has been an upsurge of interest in the application of several techniques of open quantum systems.
- These have, hitherto, concentrated on Lindbladian dynamics: Born, Markov, RW approximations.
- Due to impressive developments in the experimentation with quantum systems and their control, it has been realized that open quantum systems do not generally behave within the domain of Markovian dynamics: hence non-markovian regime needs to be probed.
- Generally, a quantitative measure, which shows monotonic behavior under Markovian dynamics is addressed and departure from that monotonicity is taken to be a signature of non-Markovianity.

- We consider a global system-environment interaction [S. Bhattacharya, SB, A. K. Pati: (2016)].
- We study the dynamics of coherence, entanglement, under the above interaction, from the perspective of deviation from monotonicity.
- We also consider the generation of quantum Fisher information by the global environmental interaction.
- We see that the Quantum Fisher information is lower bounded by a measure of quantumness, based on the non-commutativity of quantum states, and upper bounded by coherence.

Global S-E Interaction Model

[Z. Ficek and R. Tanas: (2002); SB, V. Ravishankar and R. Srikanth: (2009)]

- Hamiltonian, describing the dissipative, position dependent, interaction of two qubits with bath (modelled as a 3-D electromagnetic field (EMF)) via dipole interaction as:

$$
\begin{aligned}
H &= H_S + H_R + H_{SR} \\
&= \sum_{n=1}^{N=2} \hbar\omega_n S_n^z + \sum_{\vec{k}s} \hbar\omega_k (b^\dagger_{\vec{k}s} b_{\vec{k}s} + 1/2) \\
&- i\hbar \sum_{\vec{k}s} \sum_{n=1}^{N} [\vec{\mu}_n.\vec{g}_{\vec{k}s}(\vec{r}_n)(S_n^+ + S_n^-) b_{\vec{k}s} - h.c.].
\end{aligned}
$$

- $\vec{\mu}_n$: transition dipole moments, dependent on the different atomic positions $\vec{r}_n$

- Dipole raising and lowering operators

$$S_n^{+} = |e_n\rangle\langle g_n|, \ S_n^{-} = |g_n\rangle\langle e_n| :$$

satisfy the usual commutation relations

$$S_n^{z} = \frac{1}{2}(|e_n\rangle\langle e_n| - |g_n\rangle\langle g_n|) :$$

energy operator of the nth atom

- $b^{\dagger}_{\vec{k}s}$, $b_{\vec{k}s}$: creation and annihilation operators of the field mode (bath) $\vec{k}s$ with the wave vector $\vec{k}$, frequency ω_k and polarization index $s = 1, 2$

- System-Reservoir (S-R) coupling constant:

$$\vec{g}_{\vec{k}s}(\vec{r}_n) = (\frac{\omega_k}{2\varepsilon\hbar V})^{1/2}\vec{e}_{\vec{k}s}e^{i\vec{k}.r_n}.$$

  V: the normalization volume and $\vec{e}_{\vec{k}s}$: unit polarization vector of the field.
- S-R coupling constant: dependent on the atomic position r_n.
- This leads to a number of interesting dynamical aspects.

- Assuming separable initial conditions, and taking a trace over the bath the reduced density matrix of the qubit system in the interaction picture and in the usual Born-Markov, rotating wave approximation (RWA) is obtained as

$$\begin{aligned}\frac{d\rho}{dt} &= -\frac{i}{\hbar}[H_{\tilde{S}}, \rho] - \frac{1}{2}\sum_{i,j=1}^{2}\Gamma_{ij}[1+\tilde{N}](\rho S_i^+ S_j^- + S_i^+ S_j^- \rho - 2S_j^- \rho S_i^+) \\ &- \frac{1}{2}\sum_{i,j=1}^{2}\Gamma_{ij}\tilde{N}(\rho S_i^- S_j^+ + S_i^- S_j^+ \rho - 2S_j^+ \rho S_i^-) \\ &+ \frac{1}{2}\sum_{i,j=1}^{2}\Gamma_{ij}\tilde{M}(\rho S_i^+ S_j^+ + S_i^+ S_j^+ \rho - 2S_j^+ \rho S_i^+) \\ &+ \frac{1}{2}\sum_{i,j=1}^{2}\Gamma_{ij}\tilde{M}^*(\rho S_i^- S_j^- + S_i^- S_j^- \rho - 2S_j^- \rho S_i^-).\end{aligned}$$

$$\tilde{N} = N_{\rm th}(\cosh^2(r) + \sinh^2(r)) + \sinh^2(r),$$

$$\tilde{M} = -\frac{1}{2}\sinh(2r)e^{i\Phi}(2N_{\rm th} + 1) \equiv Re^{i\Phi(\omega_0)},$$

with

$$\omega_0 = \frac{\omega_1 + \omega_2}{2},$$

and

$$N_{\rm th} = \frac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1}.$$

Here $N_{\rm th}$ is the Planck distribution giving the number of thermal photons at the frequency ω and r, Φ are squeezing parameters. The analogous case of a thermal bath without squeezing can be obtained from the above expressions by setting these squeezing parameters to zero, while setting the temperature (T) to zero one recovers the case of the vacuum bath.In the present work we consider a simpler situation where reservoir squeezing is neglected.

$$H_{\tilde{S}} = \hbar \sum_{n=1}^{2} \omega_n S_n^z + \hbar \sum_{\substack{i,j \\ (i \neq j)}}^{2} \Omega_{ij} S_i^+ S_j^-,$$

where

$$\begin{aligned}\Omega_{ij} &= \frac{3}{4}\sqrt{\Gamma_i \Gamma_j}\left[-[1-(\hat{\mu}.\hat{r}_{ij})^2]\frac{\cos(k_0 r_{ij})}{k_0 r_{ij}} + [1-3(\hat{\mu}.\hat{r}_{ij})^2]\right.\\ &\times \left.[\frac{\sin(k_0 r_{ij})}{(k_0 r_{ij})^2} + \frac{\cos(k_0 r_{ij})}{(k_0 r_{ij})^3}]\right].\end{aligned}$$

$\hat{\mu} = \hat{\mu}_1 = \hat{\mu}_2$ and $\hat{r}_{ij}$ are unit vectors along the atomic transition dipole moments and $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$, respectively. $k_0 = \omega_0/c$, $r_{ij} = |\vec{r}_{ij}|$.

- Wavevector $k_0 = 2\pi/\lambda_0$, λ_0 being the resonant wavelength, occuring in the term $k_0 r_{ij}$ sets up a length scale into the problem depending upon the ratio r_{ij}/λ_0. This is thus the ratio between the interatomic distance and the resonant wavelength, allowing for a discussion of the dynamics in two regimes:
  (a). localized decoherence: where $k_0.r_{ij} \sim \frac{r_{ij}}{\lambda_0} \geq 1$
  and
  (b). collective decoherence: where $k_0.r_{ij} \sim \frac{r_{ij}}{\lambda_0} \to 0$.
- Collective decoherence would arise when the qubits are close enough for them to feel the bath collectively or when the bath has a long correlation length (set by the resonant wavelength λ_0) in comparison to the interqubit separation r_{ij}.

- Ω_{ij}: a collective coherent effect due to the multi-qubit interaction and is mediated via the bath through the terms $\Gamma_i = \frac{\omega_i^3 \mu_i^2}{3\pi\varepsilon\hbar c^3}$.
- The term Γ_i is present even in the case of single-qubit dissipative system bath interaction and is the spontaneous emission rate, while

$$\Gamma_{ij} = \Gamma_{ji} = \sqrt{\Gamma_i \Gamma_j} a(k_0 r_{ij}),$$

where $i \neq j$ with

$$\begin{aligned} a(k_0 r_{ij}) &= \frac{3}{2}\Bigg[[1 - (\hat{\mu}.\hat{r}_{ij})^2]\frac{\sin(k_0 r_{ij})}{k_0 r_{ij}} + [1 - 3(\hat{\mu}.\hat{r}_{ij})^2] \\ &\times [\frac{\cos(k_0 r_{ij})}{(k_0 r_{ij})^2} - \frac{\sin(k_0 r_{ij})}{(k_0 r_{ij})^3}]\Bigg]. \end{aligned}$$

- Γ_{ij}: collective incoherent effect due to the dissipative multi-qubit interaction with the bath.
- Thus, $a(k_0 r_{ij})$ characterizes the global S-E interaction: off-diagonal components of ρ get feedback from their diagonal counterparts...
- For the case of identical qubits, as considered here, $\Omega_{12} = \Omega_{21}$, $\Gamma_{12} = \Gamma_{21}$ and $\Gamma_1 = \Gamma_2 = \Gamma$.

- Coherence is one of the fundamental properties of a quantum system closely connected to quantum superposition.
- Recently, from the perspective of coherence resource theory [Baumgratz,Cramer,Plenio (2014)], it was shown that any valid measure of coherence $C(\rho)$ has the following properties:
  1. $C(\rho) = 0$ iff $\rho \in \mathcal{I}$, where $\mathcal{I}$ denotes incoherent states, which are the diagonal states in the preferred basis.
  2. Monotonicity under incoherent selective measurements : $C(\rho) \geq \sum_n p_n C(\rho_n)$. Here $\rho_n = \hat{\mathcal{K}}_n \rho \hat{\mathcal{K}}_n^\dagger$ and $p_n = Tr(\hat{\mathcal{K}}_n \rho \hat{\mathcal{K}}_n^\dagger)$ with $\sum_n \hat{\mathcal{K}}_n^\dagger \hat{\mathcal{K}}_n = \mathbb{I}$ and $\hat{\mathcal{K}}_n \mathcal{I} \hat{\mathcal{K}}_n^\dagger \subset \mathcal{I}$: coherence should not increase under incoherent operations.
  3. Convexity : $C(\sum_n p_n \rho_n) \leq \sum_n p_n C(\rho_n)$ for any set of states $\{\rho_n\}$ and probability distribution $\{P_n\}$: coherence should decrease under mixing.

- From this, after optimization over all possible incoherent states ($\mathcal{I}$), it can be shown that a measure of coherence is

$$C_{l_1}(\rho) = \sum_{i \neq j} |\langle i|\rho|j\rangle|.$$

- For any two qubit system, concurrence [Wootters (1998)] may be explicitly calculated from the density matrix as

$$E_c(\rho) = max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\},$$

where the quantities λ_i (i=1,2,3,4) are the eigenvalues of the matrix (in decreasing order)

$$\tau = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y).$$

- Here ρ^* is the complex conjugate of the density matrix ρ in the usual computational basis.

- The Fisher information has considerable significance in statistical estimation theory, as a measure of "intrinsic accuracy" [Fisher (1925), Rao (1945), Helstrom (1976), Holevo (1982)].
- Consider $p_\theta : \theta \in \mathbb{R}$ to be a family of probability densities on $\mathbb{R}$ parametrized by θ, and we have observed samples $x_1, \cdots, x_n$; each is a random variable independently distributed according to p_θ for some fixed unknown θ. The task is to estimate this as precisely as possible by virtue of the available data.
- In this context, the statistical score function $\partial \log p_\theta(x)/\partial\theta$ is of particular importance, and the Fisher information is defined as

$$F(p_\theta) = \int_{\mathbb{R}} \left(\frac{\partial}{\partial\theta} p_\theta^{1/2}(x) \right)^2 dx = \frac{1}{4} \int_{\mathbb{R}} \left(\frac{\partial}{\partial\theta} \log p_\theta(x) \right)^2 p_\theta dx.$$

- By replacing the integration by trace, probability p_θ by density matrix ρ_θ and the logarithmic derivative $\frac{\partial}{\partial\theta}\log p_\theta$ by the symmetric logarithmic derivative L_θ, determined by

$$\frac{\partial}{\partial\theta}\rho_\theta = \frac{1}{2}(L_\theta\rho_\theta + \rho_\theta L_\theta), \qquad \theta \in \mathbb{R},$$

the Fisher information can be expressed as [Luo (2003)]

$$F(p_\theta) = \frac{1}{4}Tr(L_\theta^2\rho_\theta).$$

- In [Iyengar, Chandan, Srikanth (2013); Ferro et al. (2015)], a measure of quantumness was proposed based on the incompatibility of quantum states.
- The mutual incompatibility of two given states ρ_a and ρ_b can be quantified by twice of the Hilbert-Schmidt norm of their commutator

$$Q(\rho_a, \rho_b) = 2 \parallel [\rho_a, \rho_b] \parallel^2 = 4\,Tr((\rho_a\rho_b)^2 - \rho_a^2\rho_b^2).$$

- $Q(\rho_a, \rho_b)$ is zero only when the the two density matrices commute with each other. Given this fact, $Q(\rho_a, \rho_b)$ can be considered as a powerfull quantumness witness obeying the relation

$$0 \leq Q(\rho_a, \rho_b) \leq 1.$$

- Intuitively we can state that if two states ρ_a and ρ_b do not commute, they are quantum.

Evolution of Coherence under Global S-E Interaction

- Consider the initial state

$$\rho_w = \frac{1}{4}\begin{pmatrix} 1-x & 0 & 0 & 0 \\ 0 & x+1 & -2x & 0 \\ 0 & -2x & x+1 & 0 \\ 0 & 0 & 0 & 1-x \end{pmatrix},$$

where x is a non-zero parameter lying between $1/3$ and 1.

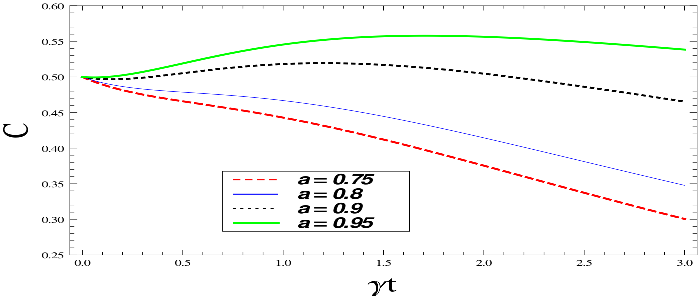

Fig. : Staring from the above state, coherence C, in the $T = 0$ regime, with respect to γt for the Werner state with $x = 1/2$, with a as a parameter. The figure shows that with increment of a, the decay of coherence slows down and as the global interaction increases, there is a generation of coherence.

Evolution of Entanglement under Global S-E Interaction

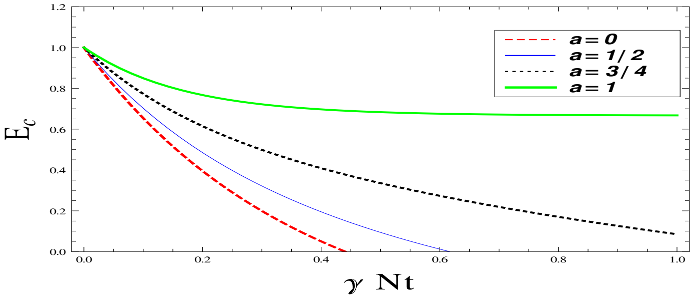

Fig. : Staring from the above state, concurrence E_C versus γNt for the Werner state with $x = 1$, with a as a parameter. The red (large dashed) plot is for $a = 0$, which is the usual Markovian case. The blue (thin line) plot is for $a = 1/2$, black (small dashed) plot is for $a = 3/4$ and green (thick line) is for $a = 1$. The figure shows that with increment of a, the sudden death of entanglement slows down, like a slow decay.

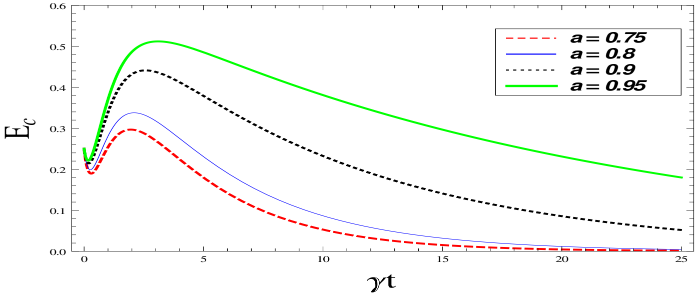

Fig. : Staring from the above state, concurrence E_C versus γt for the Werner state with $x = 1/2$, with a as a parameter. We have taken $a = 0.75, 0.8, 0.9, 0.95$ respectively. There is a region of the evolution where entanglement is generated due to the global nature of the environmental interaction.

Fisher, mutual incompatibility under Global S-E Interaction

- Consider the initial state

$$\rho_0 = \begin{pmatrix} \rho_{11}(0) & 0 & 0 & 0 \\ 0 & \rho_{22}(0) & 0 & 0 \\ 0 & 0 & \rho_{33}(0) & 0 \\ 0 & 0 & 0 & \rho_{44}(0) \end{pmatrix}.$$

- The generated quantumness and Fisher information by the action of the global operation can be expressed as

$$Q(\rho_0, \rho_t) = (\rho_{22}(0) - \rho_{33}(0))^2 C^2(\rho_0, \rho_t),$$

$$F(\rho_0, \rho_t) = \frac{(\rho_{22}(0) - \rho_{33}(0))^2}{\rho_{22}(0) + \rho_{33}(0)} \frac{C^2(\rho_0, \rho_t)}{4},$$

respectively, where $C(\rho_0, \rho_t) = 2|\rho_{23}(t)|$ is the generated coherence.

- We find the inequality

$$\frac{Q(\rho_0, \rho_t)}{4} \leq F(\rho_0, \rho_t) \leq \frac{C^2(\rho_0, \rho_t)}{4}.$$

- Since $0 \leq C(\rho_0, \rho_t) \leq 1$, the created coherence will always be greater than the created Fisher information and quantumness.

Evolution of Fisher, mutual incompatibility continued...

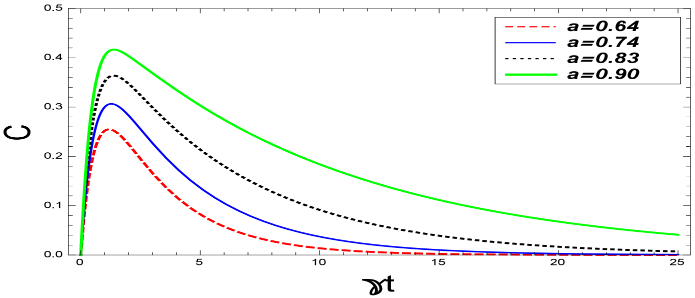

Fig. : Generated Coherence C, for zero T bath, as a function of γt. We have taken the global interaction parameter $a = 0.64, 0.74, 0.83, 0.90$, respectively. Corresponding to each value of a, we have $b (= \Omega_{12}/\gamma) = 0, -0.10, -0.24, -0.45$, respectively.

Evolution of Fisher, mutual incompatibility continued...

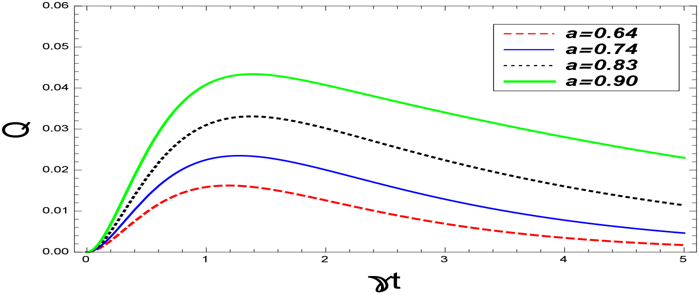

Fig. : Generated Quantumness Q, for the zero T bath, as a function of γt. We have taken the global interaction parameter $a = 0.64, 0.74, 0.83, 0.90$, respectively. Corresponding to each value of a, we have $b = 0, -0.10, -0.24, -0.45$, respectively. We see the generation of quantumness and coherence in zero temperature bath, starting from an initial diagonal state.

Global S-E Interaction with a non-Markovian Evolution

- We take up a time dependent global environmental operation based on a non-Markovian model of quantum state diffusion [Diosi,Gisin,Strunz (1998)].
- A master equation can be constructed from a non-linear stochastic Schrödinger equation of the form

$$\frac{d}{dt}\psi_t = -iH\psi_t + L\psi_t \circ z_t - \frac{1}{2}L^\dagger \int_0^t \alpha(t,s)\frac{\delta\psi_t}{\delta z_s}ds,$$

where z_t is a white complex valued Wiener process: $z_t = \sum_\nu z_\nu e^{i\nu t}$. The correlation relations are defined as

$$M[z_t^* z_s] = \alpha(t-s) \quad ; \quad M[z_t z_s] = 0.$$

Global non-Markovian S-E Interaction continued...

- $\alpha(t,s)$ is the environmental correlation function.
- $M[\cdots]$ is the ensemble mean over the classical noise z_t and the system density matrix $\rho_t = M[|\psi(t)\psi(t)|]$.
- The system Hamiltonian is taken to be $H = \frac{\omega}{2}\sigma_z$.
- The stochastic environmental influence is expressed by the Gaussian Wiener process Z_t, which drives the system through the operator L.
- Here L is chosen as $\lambda\sigma_-$. λ is a parameter describing the strength of interaction.

Global non-Markovian S-E Interaction continued...

- Assuming environmental correlation $\alpha(t,s) = \frac{\gamma_0}{2}\exp\left(-\gamma_0|t-s| + i\xi(t-s)\right)$, the reduced map for the system is

$$\begin{pmatrix} \rho_{11}(0)e^{-\int_0^t (F(s)+F^*(s))ds} & \rho_{12}(0)e^{-i\omega t - \int_0^t F(s)ds} \\ \rho_{21}(0)e^{i\omega t - \int_0^t F^*(s)ds} & 1-\rho_{11}(t) \end{pmatrix},$$

- For the case of long memory or strong coulping scenario, $\gamma_0 < 2\lambda^2$. Then

$$F(t) = \frac{\gamma_0}{2\lambda} + \frac{\sqrt{2\gamma_0\lambda^2-\gamma_0^2}}{2\lambda} \times \tan\left[\frac{t}{2}\sqrt{2\gamma_0\lambda^2-\gamma_0^2} - \tan^{-1}\left(\frac{\gamma_0}{\sqrt{2\gamma_0\lambda^2-\gamma_0^2}}\right)\right].$$

Global non-Markovian S-E Interaction continued...

- For a two qubit system with identical qubits, with the bath acting globally, the master equation is

$$\frac{d\rho}{dt} = \frac{i}{\hbar}[\rho, H_S(t)] + \sum_{i,j} \gamma_{ij}(t) \left(\sigma_i^- \rho \sigma_j^+ - \frac{1}{2}\{\sigma_i^+ \sigma_j^-, \rho\} \right),$$

with

$$H_S(t) = \sum_{i=1,2} \left(\frac{1}{2}\hbar \sigma_Z^i + \hbar \Omega_{ij}(t) \left(\sigma_i^- \sigma_j^+ + \sigma_i^+ \sigma_j^- \right) \right),$$

and

$$\Omega_{12} = \frac{3}{4}\gamma(t) b(k_0 r_{12}),$$
$$\gamma_{12} = \gamma(t) a(k_0 r_{12}),$$
$$\gamma(t) = \lambda[F(t) + F^*(t)].$$

Global non-Markovian S-E Interaction continued...

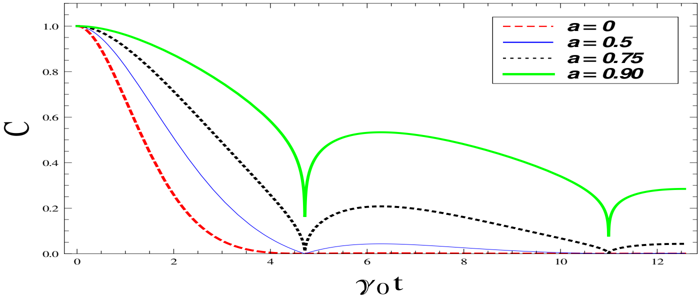

Fig. : Dynamics of coherence C for Werner state with $x = 1$, interacting with a zero T global non-Markovian bath, as a function of $\gamma_0 t$. We have taken the global interaction parameter $a = 0, 0.5, 0.75, 0.90$, respectively. We see that global environmental interaction helps the backflow of information process and as a consequence we see a periodic revival of coherence with increasing global interaction.

Global non-Markovian S-E Interaction continued...

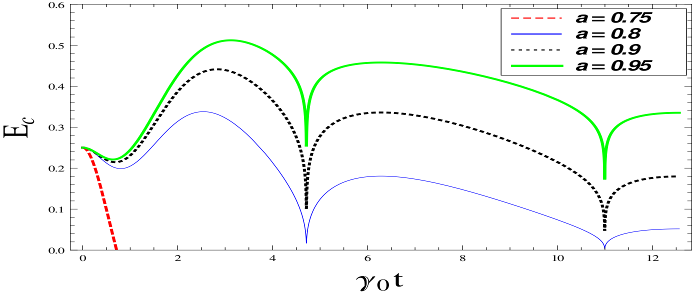

Fig. : Dynamics of entanglement E_c for Werner state with $x = 1/2$, interacting with a zero T global non-Markovian bath, as a function of $\gamma_0 t$. The global interaction parameter a is assumed to take the values $0, 0.5, 0.75, 0.90$,respectively. The global environmental interaction helps the periodic revival of entanglement.

Global non-Markovian S-E Interaction continued...

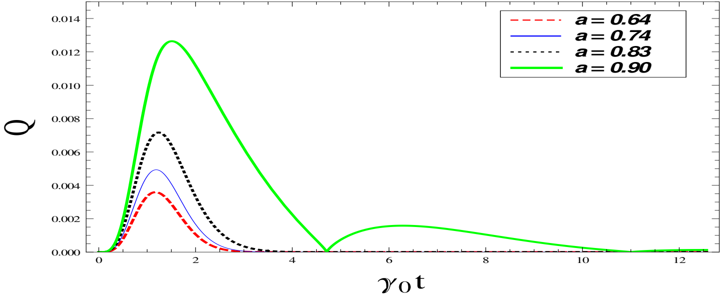

Fig. : Generated Quantumness Q, for zero T bath, as a function of γt. We have taken the global interaction parameter $a = 0.64, 0.74, 0.83, 0.90$, respectively. Corresponding to each value of a, we have $b = \Omega_{12}/2\gamma = 0, -0.10, -0.24, -0.45$, respectively. The global interaction enhances the generation of non-classicality.

Conclusions

- We exploit a useful global system-environment interaction and study the effect of non-Markovian behaviour on various facets of quantum coherence and correlations.
- The global part of the environmental interaction is acting as a resource to compensate the effect of decoherence.
- The global interaction helps the backflow of information from environment to the system via non-Markovian interaction.
- Fisher information and the quantumness measure are both based on the non-commutativity of states and this is understood as the non-classicality of quantum states.
- The generated coherence, by any arbitrary global operation, is always greater than the created quantumness and Fisher information.